A *cosine* Fourier transform of a *sine* modulated signal gives the dispersion lineshape:

$$S_0 \sin(\Omega t) \exp(-Rt) \xrightarrow{\cos \text{FT}} S_0 D(\omega).$$

A *sine* Fourier transform of a *cosine* modulated signal also gives the dispersion lineshape:

$$S_0 \cos(\Omega t) \exp(-Rt) \xrightarrow{\sin \text{FT}} S_0 D(\omega).$$

So, if we select the kind of Fourier transform which results in the cosine modulated data giving the desired absorption mode lineshape, then the sine modulated data will give peaks with the undesirable dispersion lineshape. Similarly, selecting the correct transform for the sine modulated data will result in dispersion mode lineshapes for the cosine modulated data. There is no way round this problem, although we will see in due course that it is sometimes possible to modify an experiment so as to obtain one kind of modulation.

8.2.4 Labelling the axes of two-dimensional spectra

In all of the theoretical approaches we use to analyse NMR experiments, it is generally more convenient to express any frequencies in rad s^{-1}, rather than in Hz. This is why we so far have labelled the frequency axes of our two-dimensional spectra as ω_1 and ω_2, so as to indicate that they are in angular frequency units.

However, when we are interpreting and working with practical spectra, we are certainly going to be using Hz or ppm, and not rad s^{-1}. Of course, ppm is not really a frequency scale at all, but such values can readily be converted into frequencies.

Since this chapter is concerned with using theoretical methods to predict and understand the form of two-dimensional spectra, we will label the axes ω_1 and ω_2 i.e. implying angular frequency units. This gives us a common unit for the axes and quantities such as the offsets of the two spins, Ω_1 and Ω_2. However, when we give experimental examples of two-dimensional spectra we will label the scales in ppm, as is clearly most natural.

Often, we will want to indicate that the splitting between two peaks in a two-dimensional spectrum is given by the coupling between the two spins, J_{12} (Hz). As the frequency scale is in rad s^{-1}, the splitting should be labelled $2\pi J_{12}$, so that it too will be in rad s^{-1}. Although this is technically correct, we will not do it as the result would be cumbersome and fussy. So, even though the scale is in rad s^{-1}, we will mark the splitting as J_{12}; from the context, it will always be clear what is going on.

8.3 COSY

COrrelation SpectroscopY

The COSY experiment, and its variants, is one of the most popular and useful of all two-dimensional experiments. It is a homonuclear experiment, mostly used for analysing proton spectra. From a COSY spectrum it is possible to identify the chemical shifts of spins which are scalar coupled to one another, thus enabling us to trace out the J-coupling network in the molecule.

Figure 8.7 shows a schematic COSY spectrum; broadly speaking it contains two kinds of peaks: *cross peaks*, here shown in blue, and *diagonal peaks* show in dark grey. Cross peaks have *different* frequency coordinates in ω_1 and ω_2. Such a peak appearing at frequency $\omega_1 = \Omega_A$, $\omega_2 = \Omega_B$ shows that a spin at offset (chemical shift) Ω_A is coupled to another spin at offset (chemical shift) Ω_B. Thus, the spectrum in Fig. 8.7 shows the presence of the following couplings: *A*–*B*, *B*–*D* and *C*–*E*. Diagonal peaks have the *same* frequency coordinates in ω_1 and ω_2, and are centred at the offset (chemical shift) of each spin. These peaks do not convey any particular information about the connectivity of the spins, but serve to locate the shifts in the spectrum.

We will see shortly that the 'blobs' used in Fig. 8.7 to represent cross and diagonal peaks are not single lines but collections of peaks which form a two-dimensional multiplet. So, to be more precise we should refer to *cross-peak multiplets* and *diagonal-peak multiplets*.

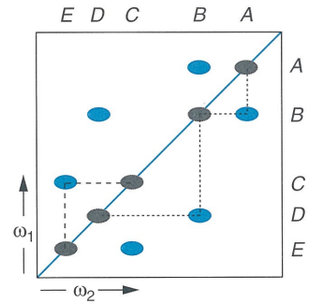

Fig. 8.7 A schematic COSY spectrum, indicating how it is used to determine the chemical shifts of coupled spins. In this example there are five spins, *A*–*E*, with the offsets indicated. Two kinds of peaks appear in the spectrum: *diagonal peaks*, shown in dark grey, and *cross peaks*, shown in blue. Diagonal peaks have the same frequency coordinates (chemical shifts) in each dimension, whereas for cross peaks the coordinates are different. The appearance of the cross peak at the ω_1 frequency of *B* and the ω_2 frequency of *A* indicates that *A* and *B* are coupled. Using the same interpretation for the other cross peaks, we find that *B* is further coupled to *D* (connections indicated by the dashed lines in the lower triangle). Similarly, *C* is coupled to *E*, but not to any of the other spins (the dashed lines in the upper triangle show this connection). Overall, the COSY spectrum allows us to trace out the network of coupled spins in the molecule. Note that the spectrum has symmetry about the diagonal, shown by the blue line.

8.3.1 Overall form of the COSY spectrum

The pulse sequence for the COSY experiment in shown in Fig. 8.8 on the following page. Although the pulse sequence is very simple, working out the detailed form of the spectrum is quite an involved process, so we will go through the calculation slowly.

We will start with equilibrium magnetization on spin one, $\hat{I}_{1z}$, which is rotated to $-\hat{I}_{1y}$ by the first 90° pulse. During t_1 this magnetization evolves under the offset of spin one and the coupling between the two spins. Evolution of the offset gives:

$$-\hat{I}_{1y} \xrightarrow{\Omega_1 t_1 \hat{I}_{1z}} -\cos(\Omega_1 t_1)\, \hat{I}_{1y} + \sin(\Omega_1 t_1)\, \hat{I}_{1x}.$$

Each term evolves under the coupling to give :

$$-\cos(\Omega_1 t_1)\, \hat{I}_{1y} \xrightarrow{2\pi J_{12} t_1 \hat{I}_{1z} \hat{I}_{2z}}$$
$$-\cos(\pi J_{12} t_1) \cos(\Omega_1 t_1)\, \hat{I}_{1y} + \sin(\pi J_{12} t_1) \cos(\Omega_1 t_1)\, 2\hat{I}_{1x}\hat{I}_{2z};$$

$$\sin(\Omega_1 t_1)\, \hat{I}_{1x} \xrightarrow{2\pi J_{12} t_1 \hat{I}_{1z} \hat{I}_{2z}}$$
$$\cos(\pi J_{12} t_1) \sin(\Omega_1 t_1)\, \hat{I}_{1x} + \sin(\pi J_{12} t_1) \sin(\Omega_1 t_1)\, 2\hat{I}_{1y}\hat{I}_{2z}.$$

Finally, each of these four terms is rotated by the second 90° pulse:

$$-\cos(\pi J_{12} t_1) \cos(\Omega_1 t_1)\, \hat{I}_{1y} \xrightarrow{(\pi/2)(\hat{I}_{1x}+\hat{I}_{2x})} -\cos(\pi J_{12} t_1) \cos(\Omega_1 t_1)\, \hat{I}_{1z} \quad [1]$$

$$\sin(\pi J_{12} t_1) \cos(\Omega_1 t_1)\, 2\hat{I}_{1x}\hat{I}_{2z} \xrightarrow{(\pi/2)(\hat{I}_{1x}+\hat{I}_{2x})} -\sin(\pi J_{12} t_1) \cos(\Omega_1 t_1)\, 2\hat{I}_{1x}\hat{I}_{2y} \quad [2]$$

$$\cos(\pi J_{12} t_1) \sin(\Omega_1 t_1)\, \hat{I}_{1x} \xrightarrow{(\pi/2)(\hat{I}_{1x}+\hat{I}_{2x})} \cos(\pi J_{12} t_1) \sin(\Omega_1 t_1)\, \hat{I}_{1x} \quad [3]$$

$$\sin(\pi J_{12} t_1) \sin(\Omega_1 t_1)\, 2\hat{I}_{1y}\hat{I}_{2z} \xrightarrow{(\pi/2)(\hat{I}_{1x}+\hat{I}_{2x})} -\sin(\pi J_{12} t_1) \sin(\Omega_1 t_1)\, 2\hat{I}_{1z}\hat{I}_{2y}. \quad [4]$$

This brings us to the start of t_2, so from now on we need only consider the observable terms, which are [3] and [4]. Term [1] represents

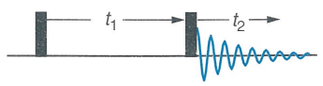

Fig. 8.8 The pulse sequence for the COSY experiment. Filled in rectangles indicate 90° pulses; unless otherwise indicated, it is assumed that the phase of the pulses is x.

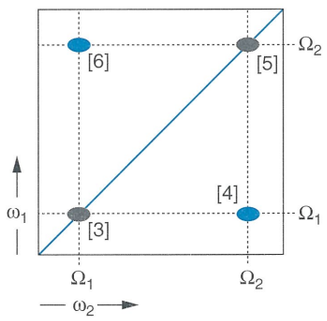

Fig. 8.9 Schematic COSY spectrum for two-spin system. There are two diagonal-peak multiplets, shown in dark grey, centred at $\{\omega_1, \omega_2\} = \{\Omega_1, \Omega_1\}$ and $\{\Omega_2, \Omega_2\}$. In addition, there are two cross-peak multiplets, shown in blue, centred at $\{\Omega_1, \Omega_2\}$ and $\{\Omega_2, \Omega_1\}$ (the internal structure of the multiplets is not shown). The numbers in square braces refer to the terms in the calculation which give rise to each feature. If the coupling between the spins goes to zero, the cross-peak multiplets disappear.

z-magnetization, and term [2] is multiple-quantum coherence, neither of which is observable.

The operator in term [3] is $\hat{I}_{1x}$, which will give rise to a doublet on spin one in the ω_2 dimension. This term is modulated in t_1 by $\sin(\Omega_1 t_1)$ i.e. it is modulated at the offset of spin one, Ω_1. Thus, in the two-dimensional spectrum, term [3] gives rise to a feature centred at the offset of spin one in ω_2, and the offset of spin one in ω_1: in other words, a diagonal peak (or more precisely a diagonal-peak multiplet). The position of the peak is shown in Fig. 8.9.

In contrast, the operator in term [4] is $2\hat{I}_{1z}\hat{I}_{2y}$; this gives rise to an anti-phase doublet centred at the shift of spin *two* in the ω_2 dimension. Like term [3], [4] is also modulated in t_1 according to $\sin(\Omega_1 t_1)$. So, overall, term [4] gives rise to a feature centred at Ω_1 in ω_1 and Ω_2 in ω_2; this is a cross-peak multiplet. Once again, the position of the peak is shown in Fig. 8.9.

We started the calculation with equilibrium magnetization on spin one, $\hat{I}_{1z}$. If we repeat the calculation starting with equilibrium magnetization on spin two, $\hat{I}_{2z}$, the resulting observable terms are

$$\cos(\pi J_{12} t_1)\sin(\Omega_2 t_1)\,\hat{I}_{2x} \qquad [5]$$
$$-\sin(\pi J_{12} t_1)\sin(\Omega_2 t_1)\,2\hat{I}_{1y}\hat{I}_{2z}. \qquad [6]$$

Using the same interpretation as above, term [5] is the diagonal-peak multiplet centred at Ω_2 in each dimension, and [6] is the cross-peak multiplet centred at Ω_2 in ω_1 and Ω_1 in ω_2. So, the complete COSY spectrum consists of two diagonal-peak multiplets and two cross-peak multiplets, as shown schematically in Fig. 8.9.

Looking back through the calculation we can see that the cross peak, term [4], arises from magnetization on spin one which went anti-phase during t_1 and was then transferred to spin two by the second 90° pulse. In other words, the cross peaks arise due to coherence transfer via the coupling.

If the coupling J_{12} is zero, no such anti-phase magnetization is generated, and so there is no cross peak. We can see this from the calculation as term [4],

$$-\sin(\pi J_{12} t_1)\sin(\Omega_1 t_1)\,2\hat{I}_{1z}\hat{I}_{2y}, \qquad [4]$$

will be zero if $J_{12} = 0$ as this makes $\sin(\pi J_{12} t_1) = 0$.

The next task is to work out the detailed form of the two-dimensional multiplets. We will find that each consists of four separate peaks, but that the phase and sign of the four peaks is significantly different between the cross- and diagonal-peak multiplets.

8.3.2 Detailed form of the two-dimensional multiplets

The cross-peak multiplet

The cross-peak multiplet arises from term [4]:

$$-\sin(\pi J_{12} t_1)\sin(\Omega_1 t_1)\,2\hat{I}_{1z}\hat{I}_{2y}. \qquad [4]$$

As was shown in section 7.5.2 on page 154, evolution of the term $2\hat{I}_{1z}\hat{I}_{2y}$ during t_2 gives rise to a time-domain signal of the form

$$\tfrac{1}{2}\mathrm{i}\exp(\mathrm{i}[\Omega_2 t_2 + \pi J_{12} t_2]) - \tfrac{1}{2}\mathrm{i}\exp(\mathrm{i}[\Omega_2 t_2 - \pi J_{12} t_2]).$$

As expected, we have terms oscillating at $(\Omega_2 + \pi J_{12})$ and $(\Omega_2 - \pi J_{12})$, which are the frequencies of the two lines of the spin-two multiplet.

If we impose an exponential decay on this time-domain signal, we obtain

$$\begin{aligned}&\tfrac{1}{2}\mathrm{i}\exp(\mathrm{i}[\Omega_2 t_2 + \pi J_{12} t_2])\exp(-R^{(2)} t_2)\\ &-\tfrac{1}{2}\mathrm{i}\exp(\mathrm{i}[\Omega_2 t_2 - \pi J_{12} t_2])\exp(-R^{(2)} t_2).\end{aligned} \qquad (8.1)$$

The Fourier transform of an exponentially decaying oscillation gives the usual spectrum, with the absorption mode in the real part:

$$S_0\exp(\mathrm{i}\Omega t_2)\exp\left(-R^{(2)} t_2\right) \xrightarrow{\mathrm{FT}} S_0\,[A_2(\Omega) + \mathrm{i}D_2(\Omega)]\,,$$

so Fourier transformation of the time-domain signal in Eq. 8.1 gives:

$$\begin{aligned}&\tfrac{1}{2}\mathrm{i}\,[A_2(\Omega_2 + \pi J_{12}) + \mathrm{i}D_2(\Omega_2 + \pi J_{12})]\\ &-\tfrac{1}{2}\mathrm{i}\,[A_2(\Omega_2 - \pi J_{12}) + \mathrm{i}D_2(\Omega_2 - \pi J_{12})]\,.\end{aligned} \qquad (8.2)$$

As expected, there are two peaks, one at $(\Omega_2 + \pi J_{12})$ and one at $(\Omega_2 - \pi J_{12})$. Most importantly, the peak at $(\Omega_2 + \pi J_{12})$ is positive whereas that at $(\Omega_2 - \pi J_{12})$ is negative, so what we have is an anti-phase doublet, just as expected for the term $2\hat{I}_{1z}\hat{I}_{2y}$.

Due to the factor of $\frac{1}{2}\mathrm{i}$ which is multiplying the whole expression in Eq. 8.2, the desirable absorption mode lineshape appears in the *imaginary* part of the spectrum. The normal practice is to adjust the phase of the spectrum so that the absorption mode lineshape appears in the real part. In this case the required phase correction is $-90°$ or $(-\pi/2)$ radians; such a correction is achieved by multiplying by $\exp(-\mathrm{i}[\pi/2]) \equiv -\mathrm{i}$. Noting that $\mathrm{i} \times (-\mathrm{i}) = 1$, the result of applying this phase correction to Eq. 8.2 is

$$\begin{aligned}&\tfrac{1}{2}\,[A_2(\Omega_2 + \pi J_{12}) + \mathrm{i}D_2(\Omega_2 + \pi J_{12})]\\ &-\tfrac{1}{2}\,[A_2(\Omega_2 - \pi J_{12}) + \mathrm{i}D_2(\Omega_2 - \pi J_{12})]\,.\end{aligned} \qquad (8.3)$$

Now the absorption mode lineshape appears in the real part of the spectrum.

For term [4], the modulation with respect to t_1 is of the form $-\sin(\pi J_{12} t_1)\sin(\Omega_1 t_1)$. This product of two sine terms can be expanded using the trigonometric identity

$$\sin A \sin B \equiv \tfrac{1}{2}\,[\cos(A - B) - \cos(A + B)]$$

to give

$$\tfrac{1}{2}\,[\cos(\Omega_1 t_1 + \pi J_{12} t_1) - \cos(\Omega_1 t_1 - \pi J_{12} t_1)]\,.$$

As before, we impose an exponential decay to give

$$\tfrac{1}{2}\left[\cos(\Omega_1 t_1 + \pi J_{12} t_1)\exp(-R^{(1)} t_1) - \cos(\Omega_1 t_1 - \pi J_{12} t_1)\exp(-R^{(1)} t_1)\right].$$